

Open Problems

Electronic Scottish Café

Problem #1

Paul Erdős

Graph Theory

(Erdős). Is it true that for every k there is a graph G , every vertex of which has degree $\geq k$ and which has no cycle of length 2^r (i.e., no cycle of length 4, 8, 16, ...)? We have no such graph for $k = 3$.

Mihol and I conjectured long ago that if G has infinite chromatic number, then for infinitely many r , G has a cycle of length 2^r .

Problem #2

Paul Erdős

Let X_1, \dots, X_n be n points in the plane. Denote by $f_k(n)$ the maximum number of distinct lines which contain $\geq k$ of our points. Determine $f_k(n)$ as exactly as you can. Trivially $f_2(n) = \binom{n}{2}$. $f_3(n) = \binom{n}{2} - cn$ (The Ehrhard Problem, *Geometriae Dedicata* 1974, by Grünbaum Sloane). $f_3(n)$ goes back at least to Sylvester. As far as I know, the value of $\lim_{n \rightarrow \infty} f_4(n)/n^2 = \lambda_4$ is not known. Is it true that $f_4(n)$ is maximal for the lattice points? It might be of interest to determine $f_4(n)$ for small values of n .

Problem #3
Paul Erdős

Let $a_1 < a_2 < \dots < a_n$ be any set of n integers. Is it true that there is a sum free subset of $n(\frac{1}{3} + \varepsilon)$ integers? I.e.,

$$a_{i_1}, a_{i_2}, \dots, a_{i_k}, k > \frac{n}{3} + \varepsilon n, a_{i_{j_1}} + a_{i_{j_2}} \neq a_{i_{j_3}}.$$

I proved it with $\frac{n}{3}$.

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