Ulam Quarterly — Volume 2, Number 2, 1993

Open Problems

Electronic Scottish Café

Problem #1
Paul Erdős
Graph Theory

(-Erdős). Is it true that for every k there is a graph G, every vertex of which has degree $\geq k$ and which has no cycle of length 2^r (i.e., no cycle of length $4, 8, 16, \ldots$)? We have no such graph for $k = 3$.

Mihol and I conjectured long ago that if G has infinite chromatic number, then for infinitely many r, G has a cycle of length 2^r.

Problem #2
Paul Erdős

Let $X_1, \ldots, X_n$ be n points in the plane. Denote by $f_k(n)$ the maximum number of distinct lines which contain $\geq k$ of our points. Determine $f_k(n)$ as exactly as you can. Trivially $f_2(n) = \binom{n}{2}$. $f_3(n) = \binom{n}{2} - cn$ (The Ehrhard Problem, Geometriae Dedicata 1974, Bu Grunbaum Sloane). $f_3(n)$ goes back at least to Sylvester. As far as I know, the value of $\lim f_4(n)|_{n^2} = \lambda_4$ is not known. Is it true that $f_4(n)$ is maximal for the lattice points? It might be of interest to determine $f_4(n)$ for small values of n.

Problem #3
Paul Erdős

Let $a_1 < a_2 < \cdots < a_n$ be any set of n integers. Is it true that there is a sum free subset of $n\left(\frac{1}{3}+\varepsilon\right)$ integers? I.e.,

$$a_{i_1}, a_{i_2}, \ldots, a_{i_k}, k > \frac{n}{3} + \varepsilon n, a_{i_{j_1}} + a_{i_{j_2}} \neq a_{i_{j_3}}.$$

I proved it with $\frac{n}{3}$.